SPRING 2025 MATH 540: QUIZ 2

Name:

1. State the uniqueness part of the Fundamental Theorem of Arithmetic (3 points)

Solution Suppose $n \in \mathbb{N}$. First version: If $n = p_1 \cdots p_r = q_1 \cdots q_s$,where each p_i, q_j is a prime number, then $r = s$ and after re-indexing the q's, $p_1 = q_1, \ldots, p_r = q_r$.

Second version: Suppose $n = p_1^{e_1} \cdots p_k^{e_k} = q_1^{f_1} \cdots q)l^{f_l}$, with p_i, q_j prime, and $e_i, f_j \geq 1$. Then $k = l$, and after re-indexing, $q_i = p_i$ and $f_i = e_i$, for $1 \leq i \leq k$. □

2. For integers, a, b, n, such that $\gcd(a, b) = 1$, prove that if $a \mid n$ and $b \mid n$, then $ab \mid n$. Hint: Use Bezout's Principle.

Solution. From the hypothesis, we can write $n = an_1$ and $n = bn_2$. By Bezout's Principle, there exist $s, t \in \mathbb{Z}$ such that $1 = sa + tb$. Thus,

$$n = san + tbn = sa(bn_2) + tb(an_1) = sn_2(ab) + tn_1(ab) = (sn_2 + tn_1)ab,$$

showing that $ab \mid n$. □

3. Find the multiplication table for remainders module 6.

Solution.

$\cdot$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

□